**Al-FARABI KAZAKH NATIONAL UNIVERSITY**

**Faculty of Mathematics and Mechanics**

**Department of Mathematical and Computer Modeling**

|  |  |
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|  | **Approve**  **at a faculty academic council meeting**  **The transactions No , 2014**  **Dean of Faculty A. B. Kydyrbekuly** |

**SYLLABUS**

**Computational solution** **of problems of the rheology**

**a second-year Magistrate, (first half-year), 3 credits, in English**

## Professor: Kanat Shakenov

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**room: 319**

**Trans-property: Probability theory, Mathematical statistics, Computational mathematics, Algebra, Geometry, Mathematical analysis, ODE, PDE, Functional analysis.**

**Post- property: Financial mathematics, Monte Carlo methods, Stochastic Process, Mathematical Modelling, Hydrodynamics, Filtration Process, Numerical Methods.**

**Goals and objectives.** We consider a process non stationary filtration flow of uniform droplet-compressible mono phase fluid in isotropic weakly-deformable porous environment. There are a various models to describe this process. The most popular is a model of classical elastic regime. But this model describes only non-stationary “equilibrium” filtration. If you take into account the fact, that the rate of flow between blocks is directly proportional to pressure difference in blocks, then this theory allows to determine a mechanism non-equilibrium filtration and obtain linear relaxation PDE for the field of pressure in blocks (PDE contains a time of relaxation). That is in the work we consider a process of non-equilibrium filtration, it is so called relaxation filtration, that take into account both non-equilibrium character of filtration law and relaxation behaviour of porosity after abrupt change in pressure. In common relaxation mechanisms can be explained by breaking of equilibrium correspondence between filtration velocity and pressure-gradient. Important feature of relaxation filtration is an existence of movable surfaces in flow. The main hydrodynamic parameters and their derivatives have a discontinuity effect (pressure, filtration velocity). This takes place with instantaneous pressure disturbances or filtration velocity disturbances and defined by relaxation time. Relaxation filtration mathematical model depends on two time functions: relaxation kernel of filtration law and fluid mass kernel. Choosing these kernels you can obtain different mathematical models of relaxation filtration. The mathematical models of nonequilibrium filtration processessolved by Monte Carlo methods.

**The structure of the course.**

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| --- | --- | --- | --- |
| **Weeks** | **Name of subject (theme)** | **duration** | **Students self-instruction (SSI) by subject** |
| **Module 1. Physical Models of nonequilibrium filtration processes in elastic porous environment** | | | |
| 1 | **Lecture 1-2.** Theory and underlying principles and equations of relaxation filtration.  **Seminar 1.** Setting of a mathematical problem. Algorithms of solution by Monte Carlo and probability difference methods. | 2  1 | SSI-1  Non-stationary equilibrium and nonequilibrium filtration process. |
| 2 | **Lecture 3-4.** Mathematical statement of Dirichlet, Neumann and mixed problems of filtration process models in elastic porous environment.  **Seminar 2.** Setting of mathematical problems. Solution of the problems by Monte Carlo and probability difference methods. | 2  1 | SSI-2  The model of filtration in relaxationaly-compressed environment realizable by the linear Darcy law. |
| 3 | **Lecture 5-6.** Mathematical problems for model relaxation filtration in relaxationaly-compressed environment realizable by the linear Darcy law.    **Seminar 3.** Algorithm of solution by Monte Carlo and probability difference methods. | 2  1 | SSI-3  The classical elastic behavior (regime) model of filtration. |
| 4 | **Lecture 7-8.** Mathematical problems (Dirichlet, Neumann and mixed) for classical elastic behavior model of filtration.  **Seminar 4.** Algorithm of solution by Monte Carlo and probability difference methods. | 2 | SSI-4  The filtration model by primary nonequilibrium law in elastic porous environment. |
| 5 | **Lecture 9-10.** Mathematical problems (Dirichlet, Neumann and mixed) for primary filtration model with spread of steady speed perturbation.  **Seminar 5.** Algorithm of solution by Monte Carlo and probability difference methods. | 2  1 | SSI-5  Primary filtration model with spread of steady speed perturbation. |
| 6 | **Lecture 11-12.** Closed system equations of linear relaxation filtration. 1. Law of conservation of impulse of resistance force. 2. Linearized law of conservation of fluid mass. Example.  **Seminar 6.** 1.Relaxation kernel of filtrationlaw. 2. Relaxation kernel of fluid mass. | 2  1 | SSI-6  PDE for pressure and filtration velocity. |
| 7 | **Lecture 13-14.** Closed system equations (PDE) of linear relaxation filtration. 3. Defining relationship for impulse of resistance force. 4. Defining relationship for fluid mass. Example.  **Seminar 7.** 2. Relaxation kernel of fluid mass. | 2  1 | SSI-7  PDE for pressure and filtration velocity. |
|  | **Total Control (TC) No.1 (Weeks 1 – 7 )** | 2 |  |
| **Module 2. The Solution of the Boundary Value Problem for Elliptic Equation** | | | |
| 8-10 | **Lecture 15-20.** The Solution of the Boundary Value Problem of Poisson and Helmholtz Equations by Monte Carlo and Probability Difference Methods. Green function of Helmholtz operator of the ball. Algorithms “Random walks on spheres” and “Random walks on lattices”. Continuous Markov Chains. Theorem of Variance.  **Seminar 8-10.** Unbiased Estimate of the Solution of Boundary Value Problem of Poisson and Helmholtz Equations. Modelling of Markov Chains. | 6  3 | SSI-8-10  Estimate of derivative of solution on parameter. |
| **Module 3. The Solution of the Initial Boundary Value Problem for Parabolic Equation** | | | |
| 11-13 | **Lecture 21-26.** The Solution of the Initial Boundary Value Problem of Parabolic Equation by Monte Carlo and Probability Difference Methods. Continuous Markov Chains. Theorem of Variance.  **Seminar 11-13.** Unbiased Estimate of the Solution of the Initial Boundary Value Problem of Parabolic Equation. Modelling of Markov Chains. | 6  3 | SSI-11-13  Unbiased Estimate of the Solution. |
| **Module 3. The Solution of the Initial Boundary Value Problem for Hyperbolic Equation** | | | |
| 14-15 | **Lecture 27-30.** The Solution of the Initial Boundary Value Problem of Hyperbolic Equation by Monte Carlo and Probability Difference Methods. Continuous Markov Chains. Theorem of Variance.  **Seminar 14-15.** Unbiased Estimate of the Solution of the Initial Boundary Value Problem of Hyperbolic Equation. Modelling of Markov Chains. | 4  2 | SSI-11-13  Unbiased Estimate of the Solution. |
|  | **Total Control (TC) No.2 (Weeks 8 – 15 )** | 2 |  |

**References**

**Basic:**

1. Y.M. Molokovich, P.P. Osipov. Basics of relaxation filtration theory. Proceeding of Kazan University, 1987, pages

106. (In Russian).

2. Christian P. Robert, George Casella. Monte Carlo Statistical Methods. Second Edition. Springer. 2004.

3. Harold J. Kushner. Probability Methods of Approximations in Stochastic Control and for Elliptic Equations. Academic

Press, New York – San-Francisco – London, 1977.

**Additional:**

1. I.M. Sobol’. Monte Carlo Method. Moscow, 1985. (In Russian).

2. I.M. Sobol’. Monte Carlo Numerical Methods. Moscow, Nauka, 1973. (In Russian).

3. S.M. Ermakov. Monte Carlo Methods and Adjacent Questions. Moscow, Nauka, 1975. (In Russian).

4. S.M. Ermakov, G.A. Mihailov. The Statistical Modelling. Moscow, Nauka, 1983. (In Russian).

5. K.K. Shakenov. Monte Carlo Methods and Applications. Methodical working. Almaty, KazSU, 1993. (In Russian).

6. Sh. Smagulov, K.K. Shakenov. Monte Carlo Methods in Hydrodynamic and Filtration Problems.

Publishing House “Kazakh University”, 1999. P. 270. (In Russian).

7. Kanat Shakenov. Articles.

**Control Test: twice.**

**SSI***:* **a few times.**

**Criterion of grade of the knowledge, marks in percent**

|  |  |
| --- | --- |
| *Lecture* | *15* |
| *SSI – theory* |
| *SSI – practice(seminar)* | *15* |
| *Total written examination* | *45* |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Weeks** | **Lecture** | | **Seminar** | | **SSI** | | **TC** | **TOTAL** |
|  | **No.** | **Mark** | **No.** | **Mark** | **No.** | **Mark** |  | **Mark** |
| 1 | 1 | 0,5 | 1 | 2 | 1 | 0,5 |  | 3 |
| 2 | 2 | 0,5 | 2 | 2 | 2 | 0,5 |  | 3 |
| 3 | 3 | 0,5 | 3 | 2 | 3 | 0,5 |  | 4 |
| 4 | 4 | 0,5 | 4 | 4 | 4 | 0,5 |  | 5 |
| 5 | 5 | 0,5 | 5 | 5 | 5 | 0,5 |  | 6 |
| 6 | 6 | 0,5 | 6 | 3 | 6 | 0,5 |  | 4 |
| 7 | 7 | **-** | 7 | 3 | 7 | **-** | 3 | 6 |
| **Total: Weeks 1-7** |  | **3** |  | **21** |  | **3** | **3** | **30** |
| 8 | 8 | 0,5 | 8 | 2 | 8 | 0,5 |  | 3 |
| 9 | 9 | 0,5 | 9 | 4 | 9 | 0,5 |  | 5 |
| 10 | 10 | 0,5 | 10 | 3 | 10 | 0,5 |  | 4 |
| 11 | 11 | 0,5 | 11 | 3 | 11 | 0,5 |  | 4 |
| 12 | 12 | 0,5 | 12 | 2 | 12 | 0,5 |  | 3 |
| 13 | 13 | 0,5 | 12 | 1 | 13 | 0,5 |  | 2 |
| 14 | 14 | - | 12 | 3 | 14 | - |  | 3 |
| 15 | 15 | - | 12 | 3 | 15 | - | 3 | 6 |
| **Total: Weeks 8-15** |  | **3** |  | **21** |  | **3** | **3** | **30** |
| **Total: Weeks 1-15** |  | **6** |  | **42** |  | **6** | **6** | **60** |

**The scale of mark of knowledge:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Letter symbol of mark** | **Digital of mark (GPA)** | **Mark (percent)** | **Mark on tradition system** |
| A | 4 | 95-100 | “excellent” |
| A- | 3,67 | 90-94 |
| B+ | 3,33 | 85-89 | “good” |
| B | 3 | 80-84 |
| B- | 2,67 | 75-79 |
| C+ | 2,33 | 70-74 | “satisfactory” |
| C | 2 | 65-69 |
| C- | 1,67 | 60-64 |
| D+ | 1,33 | 55-59 |
| D | 1 | 50-54 |
| F | - | 0-49 | “unsatisfactory” |
| I | - | - | “Incomplete discipline” |
| W | - | - | “Renunciation of discipline” |
| AW | - | - | “Deduction off discipline” |
| AU | - | - | “To take a discipline” |
| P/NP (Pass / No Pass) | - | 65-100/0-64 | “Pass / No Pass” |

Sitting of the chair consideration

Protocol No. , , 2014

**Acting as chief of chair M & C M,**

**PhD, Docent** **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D.B. Zhakebaev**

**Lecturer**

**Doctor of Science, Professor \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Кanat Shakenov**